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Probability assignments to dispositions in ontologies

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Abstract. We investigate how probabilities can be assigned to dispositions in ontologies, building on Popper's propensity approach. We show that if D is a disposition universal associated with a trigger T and a realization R, and d is an instance of D, then one can assign a probability to the triplets (d,T,R) and (D,T,R). These probabilities measure the causal power of dispositions, which can be defined as limits of relative frequencies of possible instances of T triggering an instance of R over a hypothetical infinite random sequence of possible instances of T satisfying certain conditions. Adopting a fallibilist methodology, these probability values can be estimated by relative frequencies in actual finite sequences.

Keywords. probability, upper ontology, universal, causal power, frequency

Introduction

Probabilistic and statistical notions are ubiquitous in the medical domain. These include *e.g.* the prevalence of a disease in a population, the sensitivity or the specificity of a medical test, or the probability for a person to develop a disease in a given timeframe. It would therefore be valuable if ontologies aiming at representing adequately medical knowledge could formalize probabilistic notions.

The OBO Foundry is to date one of the most significant attempts to build interoperable ontologies in the biomedical domain. In this context, the OGMS ontology [1] aims at supplying a general ontology for the medical domain. The question of how to represent probabilistic notions in this framework is still open. A first attempt on a related topic has been made by Röhl & Jansen [2], who analyze the non-probabilistic aspects of the notion of disposition. We will build on their work and combine it with Popper's work [3] on propensity in order to investigate the probabilistic dimension of dispositions. In particular, we will try to determine to which kind of dispositional entities a probability can be assigned: to universals, to particulars, or to both?

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1. Dispositions and propensities

Before investigating how to formalize the concept of propensity in ontologies, let us first introduce our general ontological framework, and explain the propensity account in the field of philosophy of probability.

1.1. Realizable entities and dispositions

The OBO Foundry relies on the upper-level ontology Basic Formal Ontology (BFO), which aims at formalizing the most general concepts that domain ontologies should be based on. At the most general level, BFO recognizes two different types of entities. On one hand, there are occurrents, which are extended in time (all processes, *e.g.* a dinner or a movie screening, are occurrents). On the other hand, there are continuants, which are entirely present at every time they exist. These include independent continuants, which roughly correspond to what we would imagine as objects (*e.g.* the Earth, a bottle of wine, a molecule) or object aggregates (e.g. a flock of birds); and dependent continuants, which inhere in independent continuants (*e.g.* the greenness of a leaf, the shape of the Earth). Amongst dependent continuants, BFO makes a distinction between qualities on one hand, and realizable entities on the other hand. Qualities are entities that can be described as “categorical”, meaning that they are constantly realized. For example, at any instant, a ball has a color and a shape, therefore these dependent continuants are categorical properties and must be classified as qualities. By contrast, realizable entities have two different kinds of phases: actualization phase, during which they are realized through some processes; and dormancy phases, during which they still exist in their bearer but are not realized (cf. [4]).

Dispositions belong to this family of realizable entities: a disposition borne by an object will lead to a given process (named here “realization”) when this object is introduced into certain specific circumstances (named here “trigger”). For example, according to OGMS, the disease “epilepsy” is a disposition whose realizations are epileptic seizures; even at times when he is not undergoing any epileptic seizure, an epileptic patient is still bearing an instance of such a disposition. Let us point that a disposition always has a categorical basis (cf. [5]) – that is, a set of categorical properties (qualities) underlying the disposition. For example, the categorical basis of the disease “epilepsy” is constituted by some anomalies in neural structures, which are going to lead to epileptic seizures when some trigger (e.g. a stressful episode, or a flashing light for photosensitive people) is happening.

Finally, dispositions can be divided between sure-fire dispositions (dispositions whose triggering process lead systematically to a realization – for example the disposition for a windshield to break when it is hit by a 30 tons truck) and probabilistic dispositions (dispositions whose triggering process lead to a realization with some probability – for example the disposition for a fair coin to land on heads when it is tossed). We will investigate this second kind of dispositions in this paper.

1.2. Probabilistic dispositions and interpretation of probabilities

According to Röhl & Jansen [2], a disposition attribution has the following general structure: x has a disposition D for realization R with a trigger T with a probability p. Here, R is the realization process of the disposition; and T is its triggering process. Additionally, probability is for them simply a number between 0 and 1, and a probability attribution is a function of the tuple <disposition, realization> to this interval. However, this definition leaves open two questions. First, it does not explain what it means to assign a probability to a disposition: how can we express, using only non-probabilistic concepts, the necessary and sufficient conditions for the assignment of a probability p to a disposition? This is a particular case of a classical problem in philosophy of probability, namely the task of interpreting probabilities. Several theories have been proposed in the past, including frequentist theories (proposed by Von Mises and Reichenbach), which interpret the probability of an event as the relative frequency of this event in a hypothetical infinite sequence of trials; logicist theories (by Keynes and Carnap), which see probabilities as degrees of entailment between two propositions; subjectivist theories (by Ramsey and De Finetti), for whom a probability is a degree of belief of a rational agent in a proposition; and finally propensity theories (by Popper), that will be detailed thereafter. The second question left open by this definition is the following: are the entities present in the tuple <disposition, realization> particulars or universals? As we will see, we will have to answer the first question in order to answer the second one.

Of note, all interpretations of probability face important difficulties: see e.g. [6] and [7] for flaws in the frequentist interpretation, and see [8] for critics of the propensity theory. Moreover, Hansson [9] has pointed to the need of second-order probabilities, suggesting that a subjectivist interpretation of probability, though necessary, is likely to be not sufficient, and should be complemented with an objectivist interpretation of probability like a frequentist or propensity theory. Of these two theories, propensity theories appear to be more solid than frequentist ones (cf. [7], [10]). The underlying realist philosophy of BFO also naturally invites a propensity approach of probability; indeed, Röhl & Jansen's work [2] on sure-fire dispositions can be extended to probabilistic dispositions along the lines of this propensity theory. So let us introduce briefly this propensity approach as it has been developed in the philosophical literature, before trying to adapt it to the framework of ontologies.

1.3. Two analysis of propensity

Popper [3] has proposed the following account of propensity (refined thereafter in particular by Mellor [11] and Williamson [12]): repeatable experimental conditions (named "test") C are endowed with a disposition (named "propensity") to produce infinite hypothetical sequences of events amongst which the limit of relative frequencies of an event E would be equal to the value of the probability of E given C. For example, according to this account, an experiment of coin tosses of a symmetrical coin is endowed with a propensity which is realized when a hypothetical infinite sequence of tosses happens, by leading to a relative frequency of results "heads" of ½.

According to Popper's account, probabilities are always conditional and there does not exist any probabilities *simpliciter*: it does not make sense to speak of the probability of E, one can only deal with the probability of E *given C*. This should not be seen as a weakness of Popper's account: it may actually be a common feature shared with all other viable approaches of probability (see [13]).

Let us call "propensity$_1$" such a disposition. It is important to understand that the trigger of the propensity$_1$ is not C, but an infinite repetition of experimental conditions C; and its realization is not E, but E happening with a given limit of relative frequencies. Also, one should note that according to this account, it is certain that the event E will happen with a given limit of relative frequency if C is repeated an infinite number of times; therefore, the propensity$_1$ is not a probabilistic disposition, but a sure-fire disposition.

However, this account can seem problematic. As a matter of fact, if all propensities would be realized only during hypothetical infinite sequences of tests, there would be no point in representing them in ontologies. Indeed, in real life, we are generally not interested in hypothetical infinite sequences of tests (such as an infinite hypothetical sequence of coin tosses), but in actual and finite sequences of tests (such as a finite sequence of coin tosses).

This problem can however be easily overcome. As a matter of fact, like any dispositional property, propensity$_1$ are associated with a categorical basis – that is, a set of categorical properties that underlie the disposition. For example, the propensity$_1$ of a coin to fall on heads is associated with a categorical basis composed by some symmetry properties of the coin. But this categorical basis is also the bearer of another dispositional property that we will name here "propensity$_2$", whose trigger is not a hypothetical infinite sequence of repetitions of C (as it was for the propensity$_1$), but a unique test C. In the coin toss example, the symmetry properties of the coin will have a causal influence not only during an infinite hypothetical sequence of tosses, but also during a unique coin toss. The latter causal influence reveals a disposition to fall on heads after a unique toss, which is a propensity$_2$. This is not a sure-fire disposition, but a probabilistic disposition. As a historical note, Popper's account of propensity actually evolved during his life from a propensity$_1$ theory to a propensity$_2$ theory – although he never properly differentiated these two interpretations, and occasionally switched between one and the other without mentioning it (cf. [10]). These two accounts should however not be seen as rivals, but as complementary.

In a nutshell, we defend here the thesis that for every propensity$_1$, there is an associated propensity$_2$ (and vice versa) such that 1) a propensity$_1$ and its associated propensity$_2$ have the same categorical basis and 2) the trigger of a propensity$_1$ associated with C is an infinite sequence of repetitions of C, whereas the trigger of the propensity$_2$ associated with C is a single instance of C. We will here be interested mainly in propensities$_2$ rather than in propensities$_1$, as they are the ones which are realized (repeatedly) in finite and actual sequences of tests that we normally encounter.

Does it mean that propensities$_1$ are of no use at all? This is not the case: we actually need them, because of some insufficiencies of the propensity$_2$ account. As a matter of fact, propensity$_2$ to an event E will have a causal influence (also named "causal power") on the realization of this event when a test C happens; and it would be desirable to define probability as the intensity of this causal power. However, to our knowledge, there is currently no theory of causal powers giving a direct interpretation (using only non-probabilistic concepts) of such a probability by referring to only one test C (as expressed by Eagle [8]: "No account of partial causation has ever quantified the part-cause-of relation in the way that is required for probability."). Still, we can define the probability of a propensity$_2$ through the associated propensity$_1$, in the following way. Let us write P_2 a propensity$_2$ for an event E and a test C; and let us consider P_1 the associated propensity$_1$ that will be realized with E happening with a limit of relative frequencies p over a hypothetical infinite sequence of repetitions of C. Then we can simply define the intensity of P_2 as having this value p (see [11] for a related strategy). For example, according to this approach, a coin has a propensity$_2$ of intensity ½ to fall on heads on a unique toss if and only if it will fall on heads with a relative frequency ½ in an infinite hypothetical sequence of tosses. Therefore, this value ½ characterizes not only the relative frequency in a hypothetical infinite repetition of coin tosses, when the propensity$_1$ is realized, but also the causal power of the propensity$_2$ in a unique toss.

This provides us with a first insight into the ontology of probabilistic dispositions. We now have to specify this account in the framework of the BFO ontology, by adapting the concept of propensity$_2$ to this framework, and by introducing the distinction between universals and particulars.

2. Ontologies and probabilistic dispositions

We will use here the analysis of dispositions, bearer, trigger and realization proposed by Röhl & Jansen [2], which introduces the following relations between particulars: **has_bearer** relates a particular of disposition with its (particular) bearer; **has_realization** relates a (particular) disposition with its realization; **has_trigger$_\mathbf{D}$**, relates a disposition to its trigger; and **has_trigger$_\mathbf{R}$** relates a realization to its trigger. Röhl & Jansen also introduce the following relations between universals: *has_bearer*, *has_realization*, *has_trigger$_D$* et *has_trigger$_R$* (here we adopt the usual convention of writing in bold the relations for which one of the relata at least is a particular, and writing in italic the relations that relate only universals).

Röhl & Jansen's analysis is restricted to sure-fire dispositions. If the triggering process of a sure-fire disposition happens, then its realization process also happens: this is expressed by through Röhl & Jansen so-called "realization principle". However, the triggering process of a probabilistic disposition can happen without its realization happening. Therefore, the realization principle is not verified for probabilistic dispositions. Neither is Röhl & Jansen's following axiom: d **has_trigger$_\mathbf{D}$** t ⇔ ∃r (d **has_realization** r ∧ r **has_trigger$_\mathbf{R}$** t), for the same reason. Instead, the following weaker axiom holds true for probabilistic disposition: ∃r (d **has_realization** r ∧ r **has_trigger$_\mathbf{R}$** t) ⇒ d **has_trigger$_\mathbf{D}$** t.

In the most general case, we want to assign probabilities to triplets <disposition, trigger, realization>. We now have to investigate whether the entities that appear in this triplet are universals or particulars. For this, let us introduce D a disposition universal, X an independent continuant universal such that D *has_bearer* X, T an occurrent universal such that D *has_trigger$_D$* T, and R an occurrent universal such that D *has_realization* R. Let us consider also a particular disposition d such that d **instance_of** D, x an independent continuant such that x **instance_of** X and d **has_bearer** x, t a process such that d **has_trigger$_\mathbf{D}$** t (and therefore t **instance_of** T), and r a process such that d **has_realization** r (and therefore r **instance_of** R) and r **has_trigger$_\mathbf{R}$** t.

We will show that we can assign a probability to two different kind of triplets: (d,T,R) and (D,T,R). In order to show this, we have to find necessary and sufficient conditions for statements like "(d,T,R) has a probability p" or "(D,T,R) has a probability p", conditions which should not mention any probabilistic concept. That is, we need to reduce probabilistic assignments to non-probabilistic statements.

3. Assignment of probability to a triplet (d,*T*,*R*)

*3.1. Definition of probability of (d,*T,R*)*

In order to illustrate the meaning of a statement like "(d,*T*,*R*) has a probability *p*", let us consider a particular case. Let us name *T'* the universal process whose instances are sets of fifty white light flashes emitted by a 100 Watts bulb at a frequency of 10 Hz, seen by a person at a distance of one meter (any instance of such a repetition of fifty flashes will be abbreviated thereafter under the name 'flashing light'); *R'* the universal "epileptic seizure"; and *D'* the universal disposition such that *D'* $has\_trigger_D$ *T'* and *D' has_realization R'* (that is, *D'* is the universal disposition of having an epileptic seizure while seeing a flashing light). Finally, let us write x' the particular Mr. Dupont, a photosensitive epileptic patient; and d' the instance of *D'* such that d' **has_bearer** x' (that is, d' is the disposition of Mr. Dupont to have an epileptic seizure when seeing a flashing light). The disposition d' is associated with a categorical basis constituted by some properties of neural structures of M. Dupont. The probability p' assigned to (d',*T'*,*R'*) should then measure the causal power of these neural anomalies in triggering an epileptic seizure of Mr. Dupont during a flashing light.

This specific disposition d' is a high-level disposition; it is likely that there are many lower-level dispositions being triggered in Mr. Dupont's brain which keep on manifesting until a particular threshold is reached and M. Dupont has a fit (see [14] for a theory of dispositions along these lines). However, an ontology of medicine would focus on such a high-level disposition – not the lower-level dispositions underlying it (unless its granularity would be pushed to the neurobiological level). Therefore, we have to find a way to define the causal power assigned to the triplet (d',*T'*,*R'*).

Such a causal power can be defined according to the lines mentioned in section 1, using the fact that universals are repeatable (that is, they can be instantiated by several particulars): *p'* equals 0.2 if and only if, in every random hypothetical infinite sequence of flashing lights perceived by Mr. Dupont, the limit of relative frequency of situations causing an epileptic seizure is 0.2.

It seems that BFO does not have enough expressive power for these concepts. As a matter of fact, a hypothetical infinite sequence is composed (at least in part) by possible, non-actual entities – whereas BFO only recognizes actual particulars. In order to circumvent this problem, we will consider here an extension of BFO that recognizes also possible, non-actual particulars. The difficulties raised by this extension are out of reach of this article; and we do not claim that BFO should be permanently extended in this way (cf. our discussion in the Conclusion section). Here, we are just concerned with finding the necessary concepts to rephrase probability assignments in non-probabilistic statements, in order to determine to which kind of entities one can assign probabilities. In the remainder of this article, we will accept that particulars can be either actual or possible entities.

Let us name "sequence generated by (d,*T*)" an infinite sequence of possible particulars which are instances of *T* and triggers of d. That is, if G is a sequence generated by (d,*T*), one can write G = $(t_1, t_2, \ldots, t_n, \ldots)$ where : $\forall i \in \mathbf{N}$, t_i **instance_of** *T* ∧ d **has_trigger** t_i. Let us now write: $G_n^R = (t_i \mid i \in [1,n], \exists r_i\ r_i$ **instance_of** *R* ∧ d **has_realization** $r_i \wedge r_i$ $\mathbf{has\_trigger_R}$ $t_i)$. That is, G_n^R is the subsequence, amongst the *n* first elements of G, of the processes that will trigger a realization of d.

Let us elaborate on our former epilepsy example. In this case, a sequence G' generated by (d',*T'*) is a hypothetical infinite sequence of flashing lights perceived by Mr. Dupont, and $G'_n{}^{R'}$ is the subsequence, amongst the n first elements of G', of the flashing lights that would cause an epileptic seizure in Mr. Dupont.

The propensity theory introduces several conditions (cf. [12]), that we can formulate in our framework the following way; let Z be a set of sequences generated by (d,*T*), then:

- Z satisfies the convergence condition iff for any sequence G of Z, $\mathrm{Card}(G_n{}^R)/n$ has a finite limit as n tends to $+\infty$.
- Z satisfies the independence condition iff for any sequences G^1 and G^2 of Z, then: $\lim_{n\to+\infty}[\mathrm{Card}(G^1{}_n{}^R)/n] = \lim_{n\to+\infty}[\mathrm{Card}(G^2{}_n{}^R)/n]$
- Z satisfies the condition of Von Mises-Church randomness iff for any sequence G of Z, if a subsequence G° of G is extracted by a recursive place selection function, then $\lim_{n\to+\infty}[\mathrm{Card}(G°_n{}^R)/n] = \lim_{n\to+\infty}[\mathrm{Card}(G_n{}^R)/n]$ (see [16] for more details; this condition is introduced to exclude sequences which are not random, *e.g.* a perfectly regular alternation of coin tosses that lead respectively to heads and tails).

Unfortunately, the set of all sequences generated by (d,*T*) will not satisfy these three conditions. For example, if a fair coin were tossed an infinite number of times, it would typically fall on heads with a limiting probability of ½. However, it is possible (although highly non-typical) that it would fall on heads at every single toss of this infinite sequence; and of course, the relative frequency of the result 'heads' obtaining in this sequence (namely, 1) is not the correct value of the probability (which is ½) (see [15] for a discussion of this point). Defining precisely what is a "typical" sequence is a challenge for this kind of propensity interpretations of probability, a challenge that we will not tackle here (let us just remark that using the strong law of large numbers to solve this problem would not work, or at least not directly and easily – see e.g. [8] on this point). One possible solution that has been considered would involve using Lewis-Stalnaker semantics for counterfactuals, arguing that such non-typical sequences would not occur in any of the nearest possible worlds in which a fair coin is tossed infinitely many times (see [16]). Here, we will just accept without more discussion this notion of "typical" sequence generated by (d,*T*). As we said, all major interpretations of probability face important difficulties; defining typical sequences is precisely the major difficulty bearing on propensity interpretations. Our purpose here is not to solve this perennial problem, but to adapt the propensity interpretation to the framework of applied ontologies.

Then, if the set of typical sequences generated by (d,*T*) satisfies the three conditions of Convergence, Independence and Randomness, one can define a probability assignment in the following way: (d,*T*,*R*) has a probability p if and only if for every typical sequence G generated by (d,*T*), $\lim_{n\to+\infty}\mathrm{Card}(G_n{}^R)/n = p$.

*3.2. Determination of probability of (d,*T,R*)*

This concept of probability thus clarified, we now face another question: how can we determine the value of the probability associated with (d,*T,R*) ? As a matter of fact, in practice, we never have direct access to hypothetical infinite sequence of events. The answer to this problem is simple: although the probabilities are defined as limits of relative frequencies in infinite hypothetical sequences, we can estimate their values through relative frequencies in actual finite sequences.

Let us call "finite sequence associated with (d,T)" a finite sequence of actual instances of T which are triggers of d. Let us assume that we have recorded such a finite sequence $G^* = (t_1, t_2, \ldots, t_N)$ where: $\forall i \in [1,N]$, t_i **instance_of** $T \wedge$ d **has_trigger** t_i. Let us define then $G^{*R} = (t_i \mid i \in [1,N], \exists r_i\, r_i$ **instance_of** $R \wedge$ d **has_realization** $r_i \wedge$ r_i **has_trigger$_R$** $t_i)$. Then the value $\mathrm{Card}(G^{*R})/N$ will provide an estimate of the probability associated with (d,T,R).

For example, if we have a recording of 27 episodes of Mr. Dupont perceiving a flashing light (these episodes being separated enough in time so that there would be no cumulative effects), and that on these 27 episodes, 6 of them led to an epileptic seizure, then 6/27 is an estimate of the value of the probability associated with the disposition of Mr. Dupont to undergo an epileptic seizure while perceiving a flashing light.

Of course, the larger our sample, the more confident we can be in our probability estimate; and statistical tests can evaluate the quality of the estimation. Moreover, we know that the relative frequency in a finite sample is certainly at least slightly different from the real probability value (even if the sample is large). However, it is a potentially reliable estimate of the probability value. One has to remember here that ontologies do not claim to be true representations of the world: the methodology underlying them is fallibilist (cf. [17]). That is, ontologies may not be true, but represent our best estimation of the reality. Therefore, probability estimates could figure in them, even if these values are probably slightly different from the real probability values.

One has to notice however that this method to estimate probabilities through finite sequences is not always available. Remember the case of Mr. Dupont and its epileptic seizures. It may be the case that we cannot register several flashing lights perceived by Mr. Dupont, and therefore that we cannot register the relative frequencies of the flashing lights that lead to an epileptic seizure. It is more likely that we will have to rely on medical data obtained on a sample of several photosensitive epileptic patients, not only on Mr. Dupont. This requires the formalization of another kind of probability, not associated with a triplet of the kind (d,T,R), but of the kind (D,T,R).

4. Assignment of probability to a triplet (*D*,*T*,*R*)

*4.1. Definition of the probability of (*D,T,R*)*

We have defined above the probability that a particular photosensitive epileptic patient (for example Mr. Dupont) undergoes an epileptic seizure when seeing a flashing light. We will now propose a definition of the probability that a non-specified photosensitive epileptic patient undergoes an epileptic seizure when seeing a flashing light; that is, the probability will be associated with a triplet containing a universal disposition *D'* borne by the universal photosensitive epileptic patient *X'*, rather than with a triplet containing a particular disposition d' borne by a particular photosensitive epileptic patient x' like Mr. Dupont.

The method will be similar to the former one. First, let us call "sequence generated by (D,T)" an infinite hypothetical sequence of couple of instances of D and T, such that in every couple the first element (the instance of D) has as trigger the second one (the instance of T). That is, if H is a sequence generated by (D,T), one can write $H = ((d_1,t_1), (d_2,t_2), \ldots, (d_n,t_n),\ldots)$ where : $\forall i \in \mathbf{N}$, d_i **instance_of** $D \wedge$ t_i **instance_of** $T \wedge$

d_i **has_trigger$_\mathbf{D}$** t_i. Let us now write $H_n^R = ((d_i,t_i) \mid i\in[1,n], \exists r_i\ r_i$ **instance_of** $R \wedge d_i$ **has_realization** $r_i \wedge r_i$ **has_trigger$_\mathbf{R}$** $t_i)$. That is, H_n^R is the subsequence, amongst the n first elements of H, of the processes t_i that trigger a realization of the disposition d_i. Let us now assume that the same three conditions as before (convergence, independence, randomness) are verified by the set of typical sequences generated by (*D*,*T*). Then one can define an assignment of probability in a similar way: (*D*,*T*,*R*) has a probability p if and only if, for every typical sequence H generated by (*D*,*T*), $\lim_{n\to+\infty}\mathrm{Card}(H_n^R)/n = p$.

Elaborating on our epilepsy example, if H' is a sequence generated by (*D'*,*T'*), H' is an infinite sequence of couple of instances < epileptic disposition borne by a photosensitive patient, flashing light perceived by this patient > ; and H'^R_n is the subsequence, amongst the n first elements of H', of the pairs (d_i,t_i) such that the flashing light t_i causes an epilepsy seizure r_i in the patient bearer of the disposition d_i. The probability *p'* associated with (*D'*,*T'*,*R'*) will then be defined as the limit of relative frequencies of flashing lights leading to epileptic seizures over a hypothetical random infinite sequence of flashing lights undergone by photosensitive epileptic individuals.

*4.2 Determination of the probability of (*D,T,R*)*

This being defined, the same question as before reappears: how can we evaluate practically the probability assigned to a triplet (*D*,*T*,*R*)? The answer will be similar to the one we gave before: the actual finite relative frequencies will provide estimates of limits of infinite hypothetical relative frequencies. For example, in the epilepsy case, in order to estimate the probability that a non-specified photosensitive epileptic patient will have an epileptic seizure when seeing a flashing light, we will estimate the relative frequency of epileptic seizures that obtained in a finite sample of flashing lights perceived by a finite sample of photosensitive epileptic patients.

More generally, let us call "finite sequence associated with (*D*,*T*)" a finite sequence of couple of instances of *D* and *T*, such that in every couple the first element (the instance of *D*) has as a trigger the second element (the instance of *T*). That is, if H* is a finite sequence associated with (*D*,*T*), one can write $H^* = ((d_1,t_1), (d_2,t_2), \ldots, (d_N,t_N))$ where : $\forall i\in[1,N]$, d_i **instance_of** $D \wedge t_i$ **instance_of** $T \wedge d_i$ **has_trigger$_\mathbf{D}$** t_i. Let us define: $H^{*R} = ((d_i,t_i) \mid i\in[1,N], \exists r_i\ r_i$ **instance_of** $R \wedge d_i$ **has_realization** $r_i \wedge r_i$ **has_trigger$_\mathbf{R}$** $t_i)$. Then $\mathrm{Card}(H^{*R})/N$ will provide an estimate of the probability associated with (*D*,*T*,*R*).

For example, if we have registered 954 flashing lights undergone by different photosensitive epileptic patients, and that on these 954 situations, 113 have led to an epileptic seizure, then an estimate of the probability that a non-specified photosensitive epileptic patient has an epileptic seizure during a flashing light would be 113/954. Here again, such an estimate may not provide the true value of the associated probability, but it fits in a fallibilist representation of reality.

*4.3 Use of the probability of (*D,T,R*) in order to estimate the probability of* (d,*T*,*R*)

As we said, it is sometimes not possible to obtain a finite sequence associated with (d,*T*,*R*), and hence to obtain an estimate of the probability of (d,*T*,*R*) as indicated in 3.2. In this case, we can try to estimate, if it is available, the probability of (*D*,*T*,*R*), where *D* is a universal instantiated by d (*i.e.* d **instance_of** *D*). As a matter of fact, since d is an instance of D, the categorical basis of d may have a causal power similar (to some extent) to the causal power of the categorical basis of D; therefore, the estimate of the probability of (D,T,R) may provide us with an approximation of the probability of (d,T,R).

Of course, the more specific the universal D, the better the probability associated with (D,T,R) will approximate the probability associated with (d,T,R). For example, the probability associated with the disposition borne by the universal of photosensitive epileptic patient to have a seizure when seeing a flashing light will provide an estimate of the probability that Mr. Dupont has a seizure when seeing a flashing light; but if we know that Mr. Dupont is a 46-years-old male, then the probability associated with the disposition borne by the universal (or defined class) of a male photosensitive epileptic patient who is between 40 and 50 years old will presumably provide an even better estimate of the probability associated with the disposition borne by Mr. Dupont. This is a version of the "principle of the narrowest reference class" due to Reichenbach [18], who proposed to "proceed by considering the narrowest class for which reliable statistics can be compiled" (see also [10]).

5. Conclusion

Let us now summarize. The probability of a disposition measures the intensity of the causal power of the categorical basis of this disposition in the realization of a given process, when a given kind of triggering process happens. The value of this causal power can be identified with the limit of relative frequencies, over an infinite hypothetical repetition of possible instances of triggering processes, of these processes that triggers an instance of a realization process. Relative frequencies obtained in a finite sequence of instances of the triggering process provide estimates of the values of these probabilities. Finally, the value of the probability associated with a triplet of the kind (D,T,R) may approximate the value of the probability associated with a triplet of the kind (d,T,R), where d **instance_of** D.

We have shown that representing a hypothetical infinite sequence of instances of triggering process requires to consider not only actual, but also possible, non-actual instances. Extending BFO to include possible instances would certainly have very significant consequences. Fortunately, we do not need to change BFO in such a way. We have shown here how, with such a change, one could define an attribution of probabilities to triplets of the kind (d,T,R) or (D,T,R). This being shown, we can now work with the classical version of BFO, restricted to actual entities, and introduce probability assignments as a primitive operation on triplets of the kind (d,T,R) or (D,T,R). It was important though to investigate the foundations of probability assignments, in order to determine to which kind of triplets we can assign a probability. Without such an investigation, the meaning of the notion of probability would have remained unclear, and it would therefore have been unclear to which kind of entities (particulars or universals) we can assign probabilities[2].

This leads us now to three questions that exceed the scope of this article. First, how should probabilistic entities and probability values be represented in the ontology?

[2] We could also wish to assign probabilities not to a particular individual or to a universal of individual, but to a particular of population or to a universal of population. This raises some new questions that have been discussed by Eells [15] (pp. 45-55).

Second, how should probability assignments to triplets of the kind (d,*T*,*R*) or (*D*,*T*,*R*) be represented in the framework of ontologies, which accept only binary relations? (and how should they be made in artifacts representing ontologies, like OWL or OBO files) This problem also appears for surefire dispositions, and it has been partially investigated by Röhl & Jansen ([2]). Future investigations concerning both surefire and probabilistic dispositions will be needed in the future. And third, at which probability threshold should we assert a disposition in an ontology? Since there is no objective threshold guided by physical reality, such a threshold should be chosen by a principle of relevance: some dispositions are so weak that there is no practical interest in representing them in an ontology. But it could be expected that this threshold of relevance would depend, amongst other factors, on the field under consideration (biology, medicine, engineering…) and on the goal of the ontology (for example, do we want to avoid type I error - false positive - or type II error - false negative - when using the ontology; cf. [19] for an introduction to this problem).

Finally, let us notice that this analysis deals only with probabilities associated with dispositional entities, which are inherently causal. In the medical domain, that would include for example: the probability to get a disease in some given circumstances; the sensitivity of a test (*i.e.* the probability to have a positive result to a test if one has the disease); or its specificity (*i.e.* the probability to have a negative result to a test if one does not have the disease). However, this account does not apply to evidential, non-causal probabilities, like the positive predictive value of a test (*i.e.* the probability to have a disease if one is tested positive) or the negative predictive value of a test (*i.e.* the probability to not have the disease if one is tested negative). In such situations, the probability characterizes the evidential strength of some evidence (for example a positive or negative test) in determining if an event (for example having got the disease) happened or not in the past. Such probabilities are epistemic and do not characterize the strength of a disposition (although their values can be constrained by probability values associated to some related dispositions). Their formalization in the framework of ontologies is therefore a totally different task, and needs to be investigated in future works[3].

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[3] Such a formalization may also enable to answer one problem left open by the present article, which is how we should account for probability of dispositions when the triggering process can vary in intensity. Here, for example, we have been interested in the probability to undergo an epileptic seizure after a set of fifty light flashes whose frequency was 10 Hz. But how should the probability be formalized if the frequency of the flashing light is not specified, and we just know that it is between 1 and 20 Hz? One solution (restricting ourselves to integer values of possible light frequencies) could be the following. First, one should determine the objective probabilities to undergo an epileptic seizure after a set of fifty light flashes of 1 Hz; of 2 Hz; of 3 Hz; …; of 20 Hz. Then, one should determine the subjective probabilities that the particular flashing light we are interested in has a frequency of 1 Hz, 2 Hz, 3 Hz,…, 20 Hz. Finally, one should combine these objective and subjective probabilities in order to compute the resulting probability that the flashing light whose frequency is not specified will cause an epileptic fit.

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